Effect of Bolt Load Proportioning on the Capacity of Multiple-Bolt Composite Joints

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This paper addresses the issue of adjusting the proportion of load transmitted by each hole in a multiple-hole joint so that the joint capacity is a maximum. Specifically, two-hole-in-series joints are examined. Joint capacity is defined to be the load to cause material failure around at least one of the holes. Results indicate that when each hole transmits exactly 50% of the total load, the joint capacity is not a maximum. The material around one hole fails, whereas the material around the other hole is intact. An algorithm, which includes failure criteria, to determine the load proportion at each hole that results in material failure at each hole and, thus, maximum joint capacity is discussed. The results for three laminates with variations in joint width, hole diameter, and hole spacing are presented.

Introduction

BESIDES delamination, few problems have attracted as much attention in the study of composite materials as the problem of joining structural components. Bonded, mechanically fastened, and mechanically fastened-bonded joints have been studied. For mechanically fastened joints, the effects of the bolts or rivets on stress distributions, load capacity, and failure modes have been the primary interests. Much of the work has been done by examining the effects of a single hole in an orthotropic plate. The hole edge is subjected to a loading that represents the effects of the bolt or rivet interacting with the hole.[1-7] However, bolted or riveted joints usually involve multiple holes, and so the analysis of a single hole in a plate is somewhat of a simplification. By incorporating a so-called bypass load in the analysis, the analysis of a single hole can be used to study multiple-hole joints.[8]

Though the bypass load idea can be used to study some of the aspects of multiple-hole joints, interaction between the holes, for example, cannot be realistically studied. Because of this, some work has been done with plates with multiple holes.[9-12] In the analysis of multiple-hole joints, an important question immediately arises. With such an analysis, there is the question of what proportion of the total load to transmit to each hole in the plate. This is a particularly important question for double-lapped joints with holes in series, such as shown in Fig. 1. For such a double-lapped joint, the compliances of the inner and outer laps strongly influence how much of the total load is transmitted to each hole. If, for example, the outer laps are much stiffer than the inner lap, then in the inner lap all of the load P is transmitted to one hole. In addition, the compliance of the fastener (bolt or rivet) can be important.

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Instead of modeling the entire joint, including all laps and the fasteners, many researchers studying multiple holes have assumed that an equal proportion of the total load is transmitted to each hole and only the inner lap, or an outer lap, has been studied. This, in most cases, has been equivalent to modeling a plate with several holes in it, each hole having a load on its edge to represent the effects of the bolt transferring the load to the hole. For the two-hole joint of Fig. 1, 50% of the applied load is assumed to be transmitted to each hole of the inner lap. However, it is important to ask if in an actual application each hole really does transmit 50% of the load. Only if the compliances of the inner and outer laps are the same is the assumption correct. Even more important, it should be asked if having each hole transmit 50% of the load results in a joint with the maximum load capacity. In a two-hole-in-series joint, with equal loading on each hole, the material around one hole may fail, whereas the material around the second hole is still intact. If this is the case, then the material around the second hole is not being used efficiently. The weight of the joint is greater than it needs to be.

This paper addresses the issue of how much of the total load should be transmitted to each hole in a two-hole-in-series joint so that the material around both holes fails. Of prime importance is whether the total load capacity of the joint is higher when this type of failure occurs than when each hole transmits 50% of the total load and only the material around one hole fails. It would seem that the capacity of the joint would be higher if the material around each hole is stressed to its capacity. The key questions are: 1) How much higher? and 2) What is the load proportion required to do this? As will be seen, a 50-50 load proportion in the two-hole joint of Fig. 1 does not result in the maximum capacity for the joint. The issue studied in this paper is a specific topic in the more general area of load path control in joints. Eisenmann and Leonhardt[13] discussed the control of load path by tailoring laminate elastic properties. Garbo and Buchanan[14] and Buchanan et al.[15] studied both laminate tailoring and varying the width and thickness of the laps.

This paper will not address the issue of how to achieve a particular load proportion. This would require modeling the inner and outer laps. The analysis here will be discussed as if

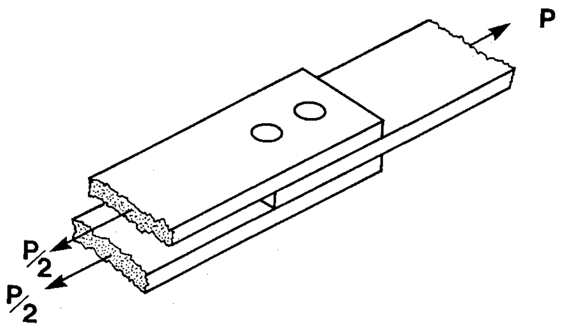


Fig. 1 Double-lapped joint with holes in series.

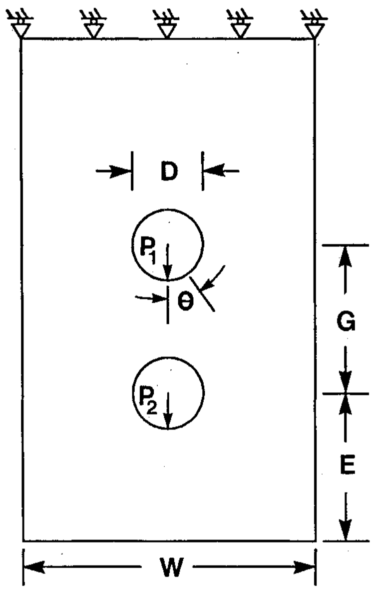


Fig. 2 Geometry and nomenclature of plate.

it is the analysis of an inner lap. It will simply be assumed that if tailoring the load proportion is beneficial, it can be done. If the outer laps are metal, for example, and the inner lap is composite, the thickness or width of the metal can be tailored as a function of position along the joint to achieve the needed compliance. Nor will this paper address tolerances between the bolts and holes, or bolt compliances. Tolerances lead to unequal loading of the holes. With care, tolerances can be controlled. Fastener compliance can be modeled and this has been discussed.[14-16] It is implicit in the present work that failure of the joint is due to failure of the material around one or both of the holes. The Yamada-Sun[17] failure criteria in conjunction with the Chang et al.[11] concept of a characteristic curve is used to predict material failure.

The paper begins by presenting the model used for the stress analysis of two-hole joints. Since the model is based on the finite-element technique, a well-known approach, the discussion is brief. Failure criteria is discussed in the next section. Following that, the algorithm is presented that is used to determine the proportion of total load that should be transmitted by each hole so that the material around each hole fails. For lack of a better term this proportion will be called the optimum load proportion. Calculation of the load capacity of the joint when using the optimum load proportion is part of this analysis. The last section presents numerical results, in tabular form, for a variety of joint geometries and three different laminates. The laminates are a quasi-isotropic $[(0/\pm45/90)_3]_s$; a cross-ply $[(0/90)_6]_s$; and an angle-ply $[(\pm45)_6]_s$. This section summarizes important conclusions based on examining the numerical results.

Model

The inner lap of a two-hole, double-lapped joint is modeled as a plate of width W with two holes, each of diameter D, a distance G apart. The plate is of thickness H. The plate and all laminae in it are assumed to be in a state of plane stress. Figure 2 shows the geometry of the plate. The distance from the bottom hole to the free end is denoted as E. Locations around the circumference of each hole are referenced by the angle θ, as denoted in the figure. The bottom of the hole is the $\theta = 0$ deg location. The nondimensional parameters W/D, G/D, and E/D will be used to describe the geometry of the plate. Because there is a hole-size effect in composites, two hole sizes will be studied. Two values each of W/D, G/D, and E/D will be investigated.

A load of P_1 is transmitted to the top hole, and a load of P_2 is transmitted to the bottom hole. The total load reacted at the fixed upper end is P, with $P_1 + P_2 = P$. Each hole is assumed to transmit its portion of the total load through radial tractions on the hole edge of the form

$$\sigma r_i(\theta) = \frac{4P_i}{\pi DH}\cos(\theta), \quad -\pi/2 \le \theta \le \pi/2 \qquad i = 1, 2$$

$$= 0, \quad \text{otherwise} \tag{1}$$

This is the cosinusoidal loading so often used in the study of isotropic materials and, to a limited extent, orthotropic materials. With this assumed form of traction at each hole, the effects of friction are ignored. The bottom and side boundaries of the plate are traction-free.

The stresses resulting from the loading and boundary conditions were determined using an eight-noded finite element. Up to 350 elements were used for discretizing one-half the plate. Around each half-hole, 20 elements were used to resolve the stresses accurately. It is assumed that the material behaves in a linear elastic fashion.

Here, fiber direction in a lamina is referenced to the horizontal direction of Fig. 2.

Failure Criteria

As mentioned in the introduction, the Yamada-Sun[17] criteria are used to predict material failure. These criteria are laminae, as opposed to laminate, failure criteria and they are applied to each lamina at each point on a so-called characteristic curve around each hole. Chang et al. introduced the concept of a characteristic curve as a generalization of the idea put forth by Whitney and Nusimer.[18] The characteristic curve is a locus of points around each hole and is given by the equation

$$r(\theta) = D/2 + R_t + (R_c - R_t)\cos(\theta) \tag{2}$$

The quantity $r(\theta)$ is the distance from the center of the hole to the locus. The quantity D is the hole diameter, and R_c and R_t are the characteristic distances of Whitney and Nuismer. The quantity R_t is the distance from the net-section hole edge ($\theta = \pi/2$) at which they applied their tension failure criteria. The quantity R_c is the distance, along the centerline ($\theta = 0$), from the bottom edge of the hole that they applied their compression (bearing) failure criteria. The characteristic curve connects these two points. Figure 3 illustrates the concept of characteristic curves around the holes in the plate. Depending on where on the characteristic curve (i.e., at which θ location) the Yamada-Sun criteria predict failure, the failure hypothesis predicts one of three failure modes: bearing, shear-out, or tension. This facet of Chang et al.'s criteria makes it a useful approach. More on this follows.

According to the Yamada-Sun[17] criteria, failure occurs at a point in a lamina if F is equal to unity at that point, F being given by

$$F^2 = \left(\frac{\sigma_1}{X}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2 \tag{3}$$

The stress σ_1 is the lamina fiber-direction stress at the point and τ_{12} is the lamina shear stress. The stresses are referred to the principal material system. The quantity X is the failure stress of the lamina in the fiber direction. If σ_1 is positive, X is the tensile failure stress in the fiber direction X_t. If σ_1 is negative, X is the compressive failure stress in the fiber direction X_c. The quantity S is the shear strength of a cross-ply laminate.[11,19] It is important to note that the stress perpendicular to the fibers, σ_2, is not involved in the failure criteria. If the stress perpendicular to the fiber controls failure, these criteria will obviously be in error. However, in a large number of cases, σ_2 is not important and so the aforementioned failure criteria are useful. The failure mode is determined as follows: If $F=1$ in the range $0 \leq \theta \leq 15$ deg, then the failure is predicted to be a bearing failure. If $F = 1$ in the range 30 deg $\leq \theta \not\leq$ deg, then the failure is predicted to be a shear-out failure. If $F=1$ in the range 75 deg$\leq \theta \leq 90$ deg, then net-section tension failure is predicted. As might be expected, if $F=1$ at intermediate values of θ, the failure is caused by a combination of modes. A failure analysis thus involves determining the value of F for each lamina at every point on the characteristic curve around each hole. The analysis is straightforward but computationally intense. Fortunately, it can be automated.

One advantage of the failure analysis used here is that the failure criteria are applied away from the edge of the hole. It is well known that a three-dimensional stress state exists at the hole edge and so a plane stress analysis is inaccurate there. However, not far from the edge a plane stress analysis can be applied. Hence, the plane stress failure criteria are applied in a region where the plane stress analysis is applicable.

Table 1 presents values of the parameters X, S, R_t, and R_c for the material used in the study. The table also presents the elastic properties of the lamina used in the laminates. The numbers are representative of T300/1034-C graphite-epoxy and are taken from Ref. 19.

Algorithm to Determine Optimum Load Proportion

Consider a plate with laminae at different orientations, for example, a quasi-isotropic laminate. This laminate has laminae at 0, 90, +45, and −45 deg. Assume that 50% of the load is transmitted at each hole. The finite-element stress analysis is conducted and the Yamada-Sun[17] failure criteria are applied on the characteristic curve around each hole. Using the stresses in each lamina and the failure data X and S, we compute the value of F as a function of location around each hole in each lamina. At some load level, in some lamina, at some θ location, and at one of the two holes, the value of F is unity. According to the Yamada-Sun criteria, the laminate fails at this load level at this θ location around this hole. The value of θ determines the mode of failure. Figure 4 shows the values of F around the two holes in the

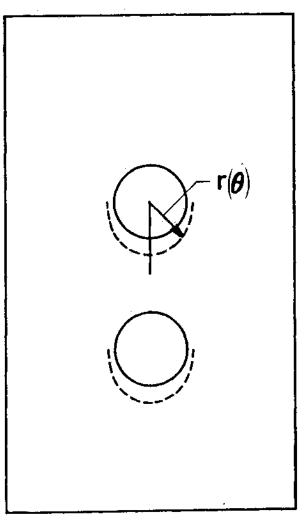
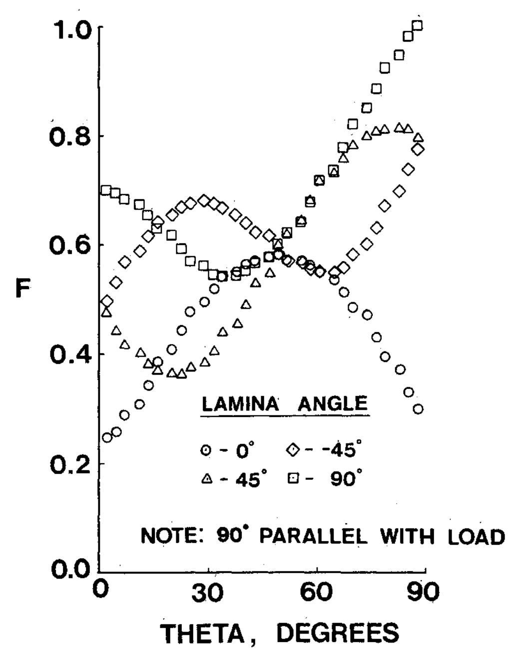


Fig. 3 Concept of a characteristic curve.

Table 1 Material properties

$E_1 = 147$ GPa	$E_2 = 11.7$ GPa	$G_{12} = 6.18$ GPa
$\nu_{12} = 0.30$	$X_t = 1.73$ MPa	$X_c = 1.38$ MPa
$S = 0.133$ MPa	$R_c = 1.778$ mm	$R_t = 0.4572$ mm



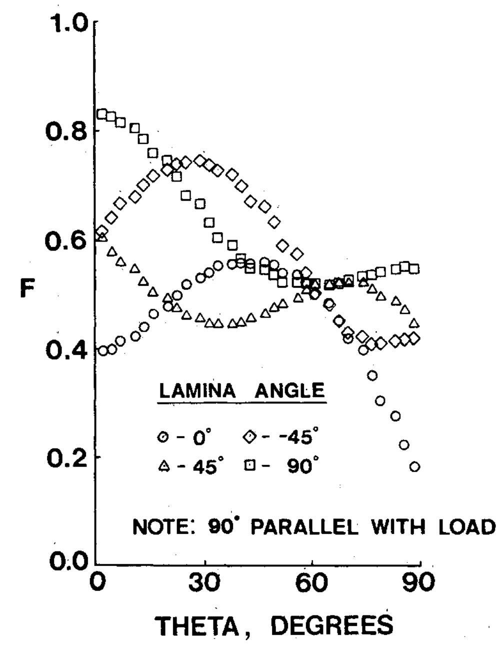


Fig. 4 F vs θ for 50-50 load proportion in a quasi-isotropic laminate: a) top hole and b) bottom hole (at failure $P/DH = 917$ MPa).

four differently oriented laminae in the quasi-isotropic laminate. Fifty percent of the total load is transmitted to each hole. The upper portion of the figure corresponds to the top hole whereas the bottom portion of the figure corresponds to the bottom hole. In each portion of the figure, there are four curves, one corresponding to the laminae in each of the four directions. The load level applied, denoted in terms of average bearing stress P/DH, is 917 MPa. This level is large enough to cause the value of F to be unity in least one location. In the figure F is unity at the net-section location ($\theta = 90$ deg) at the top hole in the 90 deg laminae, i.e., the laminae parallel to the load direction. F is not equal to unity at any other location. The next highest value of F is 0.83 at $\theta = 0$ deg laminae at the bottom hole. Therefore, in this example, the joint is predicted to fail in net-section tension due to fiber failure at the top hole.

If the net-section tensile failure at the top hole could be suppressed and the load level increased, F would eventually become unity at the $\theta = 0$ deg location at the bottom hole in the 90 deg laminae. The load level to cause this is $(1/0.83) \times 917$ MPa $= 1105$ MPa. At this hypothetical load level, the hole fails in bearing in the 90 deg laminae. The two numbers—the load to cause failure at the top hole, 917 MPa, and the load to cause failure at the bottom hole, 1105 MPa—are important and will be used shortly.

If the proportion of the total load on the top hole is decreased to 40% and then again to 30%, and the finite-element stress analysis and Yamada-Sun criteria applied to the stress computation for each to these load proportions, four more important numbers are generated. The numbers correspond to the load to cause failure at the top and bottom holes for these two other load proportions. These numbers, along with the 50% load proportion numbers, can be plotted as a function of the proportion of the total load applied to the top hole. Such a relation is shown in Fig. 5. The failure loads for the top hole, for the 30, 40, and 50% load proportion, are indicated by open circles and the failure loads for the bottom hole are indicated by open triangles. The letters beside each symbol denote the failure mode associated with that load level—B bearing failure, T net-section tension failure, and S shear-out failure. The failure loads for each hole are connected with lines. It is clear from Fig. 5 that the point where the lines for the top and bottom holes cross represents the percent load on the top hole that causes both the top and bottom holes to fail at identical load levels. In this example, the crossing of the two lines occurs when 42% of the total load is on the top hole. Furthermore, it is clear from the figure that the load capacity of the joint is higher for this load proportion than for any other proportion. In this case, the load to cause failure is 965 MPa, a 5% increase in capacity above the 50-50 case. Figure 6 shows the values of F in the 0, 90, +45, and −45 deg laminae when the optimum load proportion (42%) is used. The value of F is unity at $\theta = 90$ deg in the 90 deg laminae at the top hole and at $\theta = 0$ deg in the 90 deg laminae at the bottom hole. This indicates that, with this loading condition, the top hole fails in net-section tension, whereas the bottom hole fails in bearing.

This is the scheme used to determine the optimum load proportion and the load capacity for a variety of joint geometries and the three laminates.

Numerical Results

For the three laminates, the values of the geometric parameters studied are $W/D = 3$ and 5, $G/D = 1.5$ and 3, and $E/D = 1.5$ and 3. The two hole sizes considered are $D = 6.35$ and 12.7 cm. This represents 48 different combinations of variables. In addition, to find the optimum load proportion,

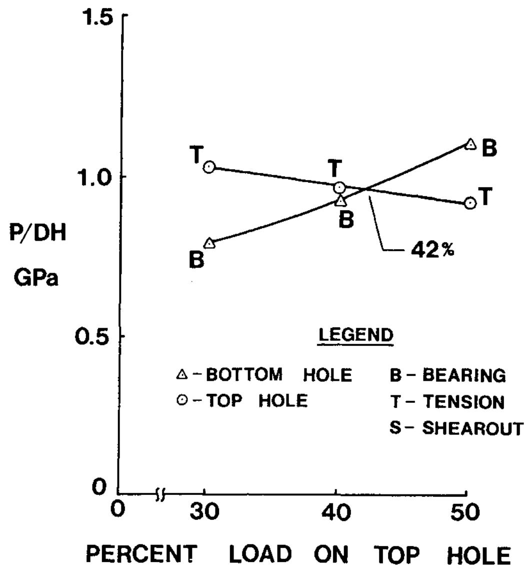


Fig. 5 Load proportion to cause failure.

Table 2 Effect of geometry on load proportioning and capacity: quasi-isotropic laminate

Case no.	Geometry W/D	E/D	G/D	D, mm	Maximum load,[a] MPa 50-50	Opt.	Optimum ratio failure mode[b] % on top hole	Top	Bottom
1	5	3	3	12.7	917	965	42	T	B
2	5	1.5	3	12.7	896	931	39	T	B
3	5	3	1.5	12.7	834	896	38	T	B
4	3	3	3	12.7	634	703	18	T	B
5	5	3	3	6.35	1103	1158	37	T	B
6	5	1.5	3	6.35	1089	1158	37	T	T
7	5	3	1.5	6.35	1000	1103	33	T	B
8	3	3	3	6.35	751	834	17	T	T
9	5	1.5	1.5	12.5	786	834	35	T	B
10	3	3	1.5	12.5	620	683	18	T	B
11	3	1.5	3	12.5	627	676	25	T	T
12	3	1.5	1.5	12.5	593	683	24	T	B
13	5	1.5	1.5	6.35	951	1076	26	T	B
14	3	3	1.5	6.35	731	820	9	T	T
15	3	1.5	3	6.35	751	800	25	T	T
16	3	1.5	1.5	6.35	703	821	18	T	T

[a]In terms of bearing stress, P/DH. [b]B = bearing, T = tension, S = shear-out.

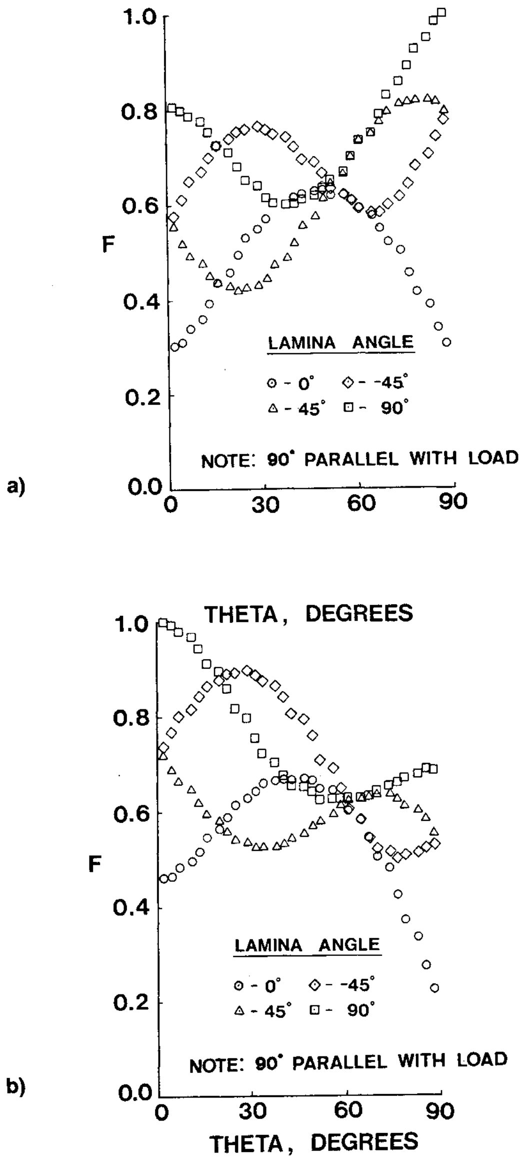


Fig. 6 ***F*** **vs** ***θ*** **for optimum load proportion in a quasi-isotropic laminate: a) top hole and b) bottom hole (at failure** ***P/DH*** **= 965 MPa).**

a minimum of three variations in load proportion must be made and a stress and failure analysis conducted. This in totality represents an enormous amount of data. The important results are summarized in tabular form in Tables 2–4. There is a table for each laminate. Each table shows the influence of the geometric variations on the load capacity. The load capacity with the 50-50 proportion and the optimum load capacity are indicated. The optimum load proportion and the failure mode at each hole when the optimum proportion is applied are indicated.

The tables will not be discussed in detail. Rather, important observations that can be made by examining the tables will be noted. These observations are the following:

1) In almost all cases, the 50-50 load proportion does not result in maximum joint capacity. Exceptions to this are cases 2 and 9, Table 4.

2) Loading the bottom hole more than the top hole almost always results in an increase in load capacity. Four cases in Table 3 are exceptions to this.

3) Generally, the increase in capacity due to changing the load proportion is not dramatic. The maximum increase in capacity is 29%. This is for case 7, Table 3.

4) The optimum load proportion is dependent on both geometry and laminate elastic properties.

5) The hole size effect is built into the scheme to determine failure and this is evident in all situations, the joints with the smaller holes sustaining larger bearing stresses than the joints with the larger holes.

6) For the quasi-isotropic material, narrowing the joint has a large influence on the load capacity, independent of the load proportion. The distance between holes and the distance of the bottom hole from the free end have less influence on capacity, for this material failure occurs by net-section tension at the top hole. Failure mode at the bottom hole depends on joint width. It is either net-section tension or bearing.

7) For the cross-ply material, load capacity is not strongly influenced by joint width. However, the capacity is influenced by the distance between the holes and by the distance of the bottom hole from the free end. The cross-ply joints always fail in shear-out.

8) The angle-ply material shows a response similar to the quasi-isotropic laminate. Narrowing the joint has the greatest effect on the reduction of load capacity. When $W/D = 5$, the angle-ply joints generally fail by net-section tension at the top hole and bearing at the bottom hole. When $W/D = 3$, these joints generally fail by net-section tension at both holes.

Table 3 Effect of geometry on load proportioning and capacity: cross-ply laminate

Case	Geometry				Maximum load,[a] MPa		Optimum ratio failure mode[b]		
no.	*W/D*	*E/D*	*G/D*	*D*, mm	50-50	Opt.	% on top hole	Top	Bottom
1	5	3	3	12.7	552	572	47	*S*	*S*
2	5	1.5	3	12.7	469	503	53	*S*	*S*
3	5	3	1.5	12.7	414	503	41	*S*	*S*
4	3	3	3	12.7	503	538	46	*S*	*S*
5	5	3	3	6.35	683	731	47	*S*	*S*
6	5	1.5	3	6.35	579	634	54	*S*	*S*
7	5	3	1.5	6.35	469	607	39	*S*	*S*
8	3	3	3	6.35	627	669	45	*S*	*S*
9	5	1.5	1.5	12.7	414	434	47	*S*	*S*
10	3	3	1.5	12.7	407	483	42	*S*	*S*
11	3	1.5	3	12.7	469	483	52	*S*	*S*
12	3	1.5	1.5	12.7	407	427	47	*S*	*S*
13	5	1.5	1.5	6.35	483	517	45	*S*	*S*
14	3	3	1.5	6.35	469	572	39	*S*	*S*
15	3	1.5	3	6.35	572	593	52	*S*	*S*
16	3	1.5	1.5	6.35	469	503	46	*S*	*S*

[a]In terms of bearing stress, *P/DH*. [b]*B* = bearing, *T* = tension, *S* = shear-out.

Table 4 Effect of geometry on load proportioning and capacity: angle-ply laminate

Case no.	Geometry				Maximum load,[a] MPa		Optimum ratio failure mode[b]		
	W/D	*E/D*	*G/D*	*D*, mm	50-50	Opt.	% on top hole	Top	Bottom
1	5	3	3	12.7	478	490	43	*T*	*B*
2	5	1.5	3	12.7	483	483	50	*T*	*B*
3	5	3	1.5	12.7	441	455	44	*T*	*B*
4	3	3	3	12.7	303	338	18	*T*	*T*
5	5	3	3	6.35	517	573	32	*T*	*B*
6	5	1.5	3	6.35	531	552	42	*T*	*B*
7	5	3	1.5	6.35	510	538	30	*T*	*B*
8	3	3	3	6.35	331	372	18	*T*	*T*
9	5	1.5	1.5	12.7	414	414	50	*T*	*B*
10	3	3	1.5	12.7	290	317	17	*T*	*T*
11	3	1.5	3	12.7	303	317	22	*T*	*T*
12	3	1.5	1.5	12.7	290	310	23	*T*	*T*
13	5	1.5	1.5	6.35	455	469	38	*T*	*B*
14	3	3	1.5	6.35	331	358	10	*T*	*B*
15	3	1.5	3	6.35	331	358	26	*T*	*T*
16	3	1.5	1.5	6.35	317	365	20	*T*	*T*

[a]In terms of bearing stress, P/DH. [b]B = bearing, T = tension, S = shear-out.

It is somewhat surprising to find that, in general, there is not much to be gained by changing the load proportion from 50-50. The primary reason for this is the insensitivity of the failure load of the top hole to the proportion of load reacted there. This can be observed in figures similar to Fig. 4. The slope of the line associated with the top hole is generally quite shallow. From a design viewpoint, it appears there is no large penalty for designing to a 50-50 capacity.

Conclusions

It should be mentioned before closing that some of the conclusions of this study depend on the assumptions used in the model, whereas other conclusions are not as sensitive to the assumptions. Here, it has been assumed that the bolts exert a cosinusoidal loading over a 180 deg arc to the hole. It is known that bolt flexibility, clearance between the bolt and hole, and friction between the bolt and hole alter the cosinusoidal loading (see, e.g., Ref. 5). The bolt flexibility, clearance, and friction no doubt will change the stress distribution around the holes, and in particular, on the characteristic curves. In addition, the location of the characteristic curves depends on the material properties R_t and R_c. Here, it has been assumed that R_t and R_c depend on the material, but not on the lamination sequence. It is safe to say, however, that while inclusion of bolt flexibility or changing the value of R_t, for example, may alter the stress magnitudes relative to the magnitudes calculated here, the proportion of the loading to each hole will not be strongly affected. Thus, the predicted failure load may change with inclusion of bolt flexibility or friction, etc., but the conclusion regarding no large penalty for designing to a 50-50 capacity will not be changed. That was the focus of the study, not absolute failure load levels.

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